**Discrete age-structured SEIR epidemic model with applications to measles vaccination strategies**

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A Research Proposal for the Biomathematics Project

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1 August 2022

**Introduction**

Measles is one of the Vaccine-Preventable Diseases (VPDs) that can cause serious disease and complications of the disease. Measles is a highly contagious aerial viral infection. Measles is an infection that causes fever, rash, cough, and redness of the eyes. It causes serious complications like encephalitis and can even lead to deafness [1]. Transmission occurs through direct contact with infectious droplets or by airborne spread when an infected person breathes, coughs, or sneezes [2]. About 90% of exposed susceptible individuals are exposed to measles. Clinically, the incubation period from exposure to early symptom onset of disease averages 10 -12 and 14 days from exposure to the rash. Serious complications caused by measles can occur in approximately 30% of measles cases, children under 5 years of age or adults [3].

Prior to the development of the measles vaccine in the 1960s, measles was a major cause of morbidity and mortality [4]. Childhood measles infections have become practically universal since the development of safe and effective vaccines in 1963, killing an estimated every year, 2.6 million people are affected. [5]. Despite the availability of vaccines, measles continues to be the leading cause of death in children under the age of five [6]. Measles outbreaks are still occurring in countries where vaccination coverage is low [7]. According to the WHO, global efforts to increase vaccination coverage lowered deaths by 73% in 2018, with Africa accounting for the majority of deaths saved. In 2012, WHO updated the Measles Eradication Initiative with the goal of eradicating measles in at least five of the six regions of the world by 2020 [8]. Measles elimination is defined by the World Health Organization as the absence of indigenous measles cases in a certain area for up to 12 months in the presence of high-quality surveillance systems. WHO also requires a national measles vaccination rate of 95% in all districts with two vaccinations per child. Within a year, at least 80% of districts should examine at least one suspicious case and report a nationwide non-measles case rate of at least 2 cases per 100,000 people. [9].

Vaccination is the most effective way to avoid measles. Single-dose measles vaccine became available in South Africa in 1975 as part of the Expanded Programme on Immunization (EPI). Single dose vaccination against measles was started in South Africa in 1975 as part of the Expanded Program on Immunization (EPI). Then, in 1995, a two-dose strategy was introduced for 9 and 18 months, with additional vaccination campaigns every 3-4 years. The two vaccination schedules for measles were altered to 6 months and 12 months in 2016. The first dose at 6 months is intended to prevent the high morbidity and mortality associated with infant illness [10-12]. To prevent the outbreak of measles, the immunity rate of the population is estimated to be about 95% [13]. In 2016, the World Health Organization (WHO) found that only 85% of children worldwide received the first dose of measles vaccine through regular medical services by the first birthday, and 64% received the second dose [14].

World Health Organization (WHO) strongly encouraged the usage of MMR vaccines to get rid of the measles virus inside the nations by enforcing large-scale immunization programs [15]. The National Institute of Communicable Diseases recommends that all children be vaccinated twice with the MMR (Measles and Mumps Rubella) vaccine. The first vaccination is scheduled for 6 months old, and the second vaccination is administered for 12 months old. Children can receive a second dose earlier if at least 28 days have passed since the first dose [22].

One of the most important contributors to population heterogeneity is age distribution, which has a significant impact on the timing and outcome of infectious transmission and spread. Most crucially, there is a considerable degree of non-uniformity in transmission rates due to the patterns and frequency of individual encounters, which can range dramatically between age groups. Age-related differences in immunity to infections are another possibility. Age-specific mortality and infection-related recovery may be impacted by these variations. Understanding the complexity of disease dynamics and implementing effective disease control and prevention depend on modelling the effect of population age composition on the spread of infectious diseases.

The age structure of epidemiological models has been studied in the literature using both discrete and continuous approaches. Discrete age Ordinary Differential Equation (ODE) Model [23, 24, 25]. The mathematical frameworks of ODE models are relatively simple due to their finite-dimensional involving properties in unaltered space, but the challenge in their theoretical analysis lies in the high dimensionality and scale of the ODE system, which is not possible. Establishing global dynamics for trendy models of age structure.

Consider a linking system of nonlinear differential equations on a transmission network as a disease model with a discrete age structure. Each age group in this situation can be thought of as a node, and inter-group transfers determine the connections between the nodes and the aging process.

In this paper, we present a discrete age structured SEIR epidemic model with application to the measles vaccination strategy. Vaccines are the most effective way to prevent infectious diseases. The measles vaccine, which is commonly administered to babies as part of the measles-mumps-rubella (MMR) vaccination, is one of the most widely used and effective vaccines. The effectiveness of a single dose of the measles vaccination administered to babies at 6 months of age ranges from 85% to 93%. And the second dose is administered at 12 months [16]. The effectiveness of two doses of the measles vaccine is 93-99%. In addition to regular vaccinations, South Africa has supplementary vaccination activities every 3-4 years. These are usually vaccination campaigns for all children under the age of five. These objectives are to immunize children who may have missed the measles vaccine and increase the effectiveness of the vaccine [26].

The World Health Organization (WHO) estimates that millions of measles cases occur in developing countries each year, primarily due to measles, even with regular immune programs [27], mainly due to the vaccination rate being too low. A discrete age structure model was fitted to analyse measles data from South Africa. Investigating the impact of current vaccination programs in South Africa. The incidence of measles is the highest in the world.

**Mathematical Model**

**An application to vaccination strategies for measles**

Measles is a disease that can be prevented with a vaccine. When given as part of the measles, mumps, rubella (MMR) vaccine, the measles vaccine usually needs to be given twice. The measles vaccine (MMR1) is normally administered to infants 12 to 15 months of age. It is effective for 85% to 93% of infants at this age. A second dose (MMR2) should be given before the child starts school. Efficacy with 2 doses can reach 97% of MMR [16,17]. World Health Organization recommends that children between 15 and 18 months of age or at admission receive MMR2 for high immunity, either to prevent measles or as a part of admission for better immunity. [18].

In this section, we develop a two-dose vaccination version with two age corporations to observe the vaccination strategies for measles epidemics.

**Measles vaccination model**

Measles can be prevented with the MMR vaccine. The CDC recommends that children receive the MMR vaccine twice. The first dose is 6 months old and the second dose is 12 months old [19]. One dose of MMR vaccine is 93% effective against measles while two doses of MMR vaccine are 97% effective against measles. A small number of vaccinated individuals can get measles, but the disease is milder than unvaccinated individuals [20].

We subdivide the host population into two age groups, considering age-specific differences in vaccination schedules, case fatality rates, and contact patterns.

The model structure is shown in the transmission diagram in Figure 1. Two doses of measles vaccine were incorporated: MMR1 for age group 1 (6 months), MMR2 for age group 2 (12 months). The model is described by the following system of differential equations.

Figure 1: Transfer diagram for a vaccination model with two age group

Differential equations for age group 1:

Differential equations for age group 2:

The model parameters are shown in Table 1 along with their description and parameters. Specifically, θ1 and θ2 are the vaccination rates of MMR1 and MMR2, respectively, σ1 and σ2 are the effects of MMR1 and MMR2, respectively, and θ1σ1 and θ1σ1 are the effective ranges of MMR1 and MMR2, respectively.

The transmission coefficient βkj between Sk and Ij is decomposed into two factors βkj = ckj where βk k is the probability of transmission for an average contact between a susceptible individual in age group k Sk, and ckj is the mean. Number of contacts between people in age group and people in age group k. Note that ckj and cjk are not the same and the contact matrix (cjj) may not be symmetric because of different ages.

Influx susceptible individuals are recruited by the rate of Λk. Exposed individuals move to the infectious class at a rate of an age group of ϵk. Infectious individuals move to the recovered compartment at a rate of an age group of γk. Individuals are aging at a rate αk. Natural fatality rate of an age group is represented by dk, while case fatality of a age group is represented by a rate of μk.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameters | Values/Range | Unit | Description | Ref |
|  | 500 |  | Influx of susceptible | [] |
|  |  |  | The natural mortality rate of age group k | [] |
|  |  |  | Aging rate of age group k | [] |
|  |  |  | Recovery rate of age group k | [] |
|  | 0.72 |  | Exposed rate of age group | [32] |
|  |  |  | Case mortality rate of age group k | [] |
|  | 0.717 |  | Vaccination coverage of Measles vaccine | [28] |
|  | 0.764 |  | Vaccination coverage of Measles vaccine | [28] |
|  | 0.93 |  | Efficacy of MMR1 | [29] |
|  | 0.95 |  | Efficacy of MMR2 | [29] |
|  |  |  | Probability of transmission per contact for age group 1 | [] |
|  |  |  | Probability of transmission per contact for age group 2 | [] |
|  |  |  |  | [] |
|  |  |  | Average number of contacts from age group j to age group k | [] |

Table 1: Parameters and their estimated values for model

**Parameters estimation and model calibration**

As indicated in Table 1, values of some parameters and initial values of state variables in model are estimated directly from published data. Other parameter values, especially those of the probability of transmission per contact and the recovery rate from measles for each age group, are estimated by fitting the model outcomes to measles data using the nonlinear least squares method [30]. The measles data used for model fitting include the reported annual incidence and age specific incidence of measles in India from 2015 to 2020 [31]. The values of measles case mortality ratio are and . By the end of 2020, the values of , are the actual vaccination rates published by NICD.

**Immune profile analysis**

In our model, we generated the measles immune profile analysis for total population and for different age group. The endemic level of measles vaccination strategies in South Africa, namely, a single-dose vaccine at 6 months old (age group 1) and the second dose vaccine at 12 months old (age group 1) during years 2015-2020. In South Africa, vaccination coverage of children under 1year averaged 71.1% , whilst measles second dose vaccination coverage is 76.4% [28]. The efficacy of two doses of measles vaccine range from to 93-99%. We therefore assume that the efficacy of the first dose is 93% and for the second dose is 95% [29].

**Effect of increasing measles and improving vaccination coverage**

Vaccination is critical to sustaining and increasing vaccination coverage rates and preventing outbreaks of measles vaccine preventable disease. The strong enforcement may help promote higher rates of vaccination coverage along with complementary actions such as monitoring VPD cases. The vaccination coverages of single-dose (MMR 1) and the second dose (MMR 2) should be both increased to 95% . In South Africa, the efficacy of MMR 1 can reaches 99% when administered to children 6 months old.

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